Topology Uses for

- **Pattern Recognition in Mineral Exploration**

&

- **Alternative Approach Road Center Lines** [1]

by Humphrey Boogaerdt

This paper is an overview of how topology can be used in pattern recognition in mineral exploration and a section about an alternative topological approach to road center lines.

What is Topology?

Topology is sometimes described as rubbersheeting, where the sheet moulds over any object underneath while the size of the sheet has not changed. A more price definition is from Wikipedia "*In mathematics, topology is concerned with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, crumpling, and bending; that is, without closing holes, opening holes, tearing, gluing, or passing through itself.* "

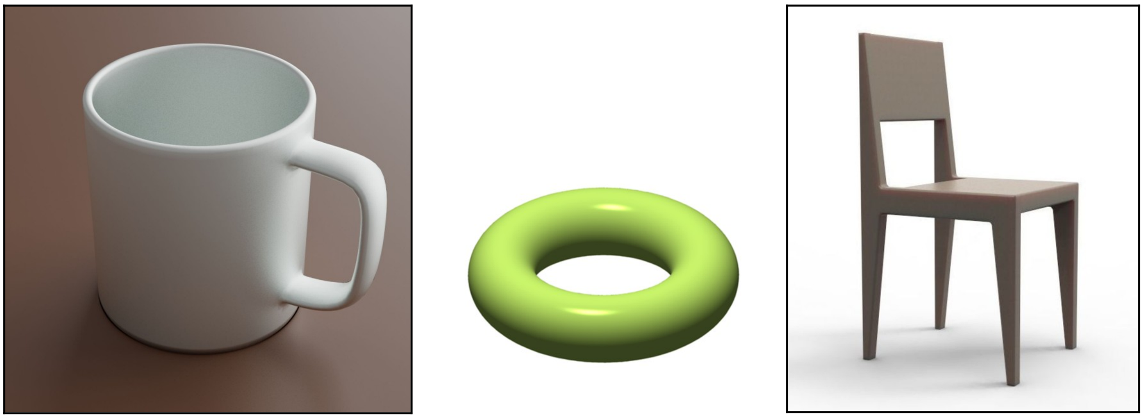
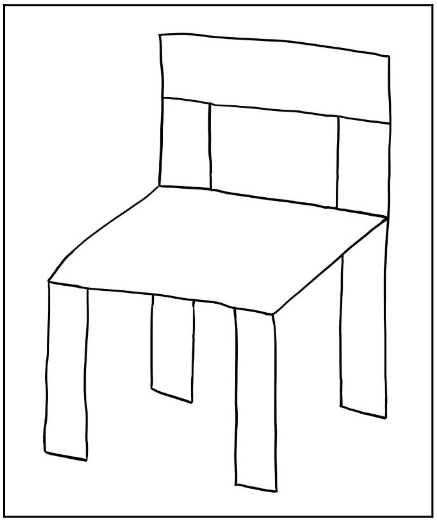
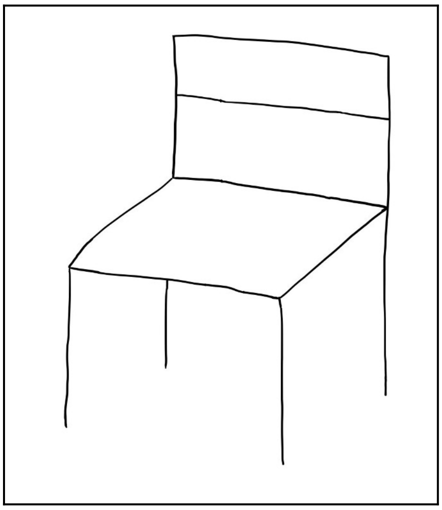


Figure-1. The mug and the chair are topologically the same.

For simplicity in this essay we deal with 2D space only even for a 3D object like a chair. The chair and the mug in figure-1 are topologically the same because their shape can be modified from one to the other without breaking the surface. They are both a Torus, and commonly known as a donut.

1 This is the paper from which the presentation was created for the Geogeeks meeting in Perth, 14 September 2023.





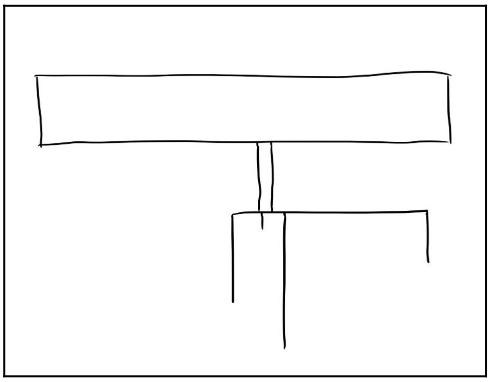


Figure-2a : Simple chair

Figure-2b : Simplificatio of a simple chair

Figure-2c :Caricature of a simple chair

Redrawn from Boogaerdt (1996)

In figure-2a there is a simple chair, it is then schematised to simple lines in figure-2b and in figure-2c a caricature of the same chair is drawn. During the morphing the topology of this chair has not changed. The chair has vertices / nodes / centroids added in figure-3 on the drawing from fig-2b to identify each element of the chair in. The parts of the chair, polylines and polygons, numbered points representing the different topological entities. On the right these relationships are put in to a graph form.

Graph theory is extensively used in network analysis. Platforms like Google and Uber will be using it for creating routes. Most results will be based on approximations, because most problems are intractable due to the number of nodes and possibilities. The “travelling salesman problem” an example is probably the first one popularised and documented in the 1930s (Wiki, 2023-a). The first notion of the problem was formulated by the mathematician Euler in 1736 about the “seven bridges problem in Königsberg” (Wiki, 2023-b).

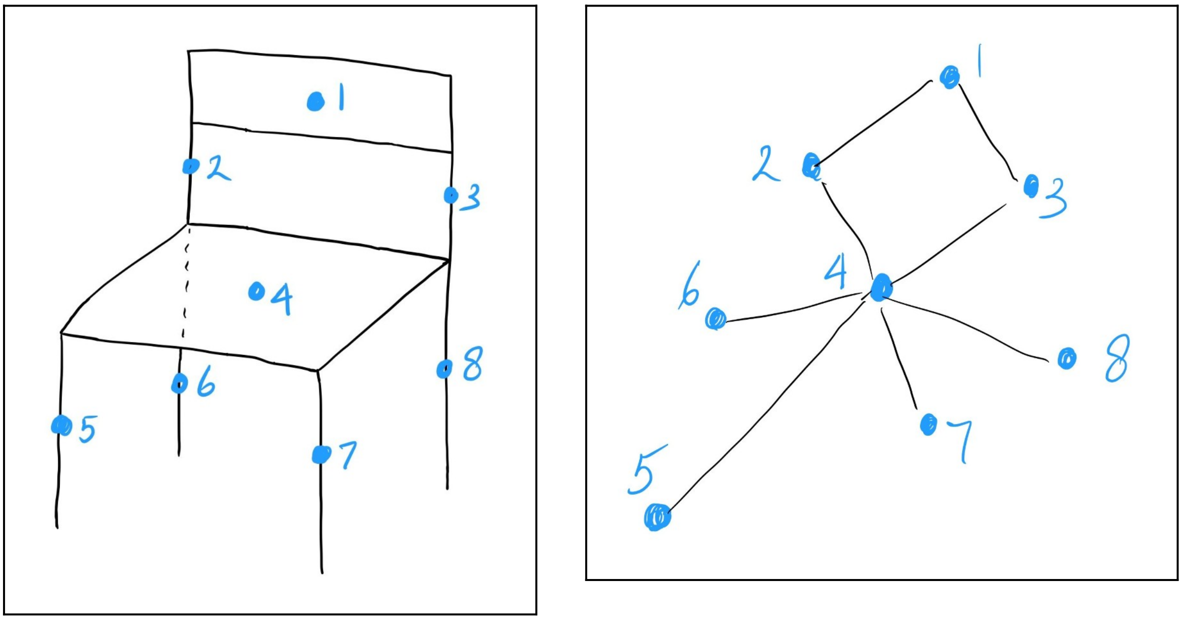


Figure 3. The numbered points in the left drawing represent the the topological entities. On the right the relationships graph.

Even though geometry and geographic position are based on physical positions and shapes, the objects are constrained by their topological relationships. In GIS topology is often used to validate the digitised or imported objects for their relationships and is also used when carrying out spatial queries.

Graph Theory and Topology are both branches of mathematics. Because topology has no dimension only relationships that can be represented as graphs. By their nature graphs embed and represent topological relationships.

A requirement for a polygon is that it cannot have crossing boundaries (fig. 4). We talk here about in the 2D plane. A 2D view of polygon in 3D space can appear to have crossing boundaries. In this case a “twisted” polygon is really a “surface” in 3D space. In GIS uses different file structures for points, lines or polygons, since these entities have different dimensions 0, 1 and 2 respectively.
To distinguish between these object files it is suggested to use a suffix to indicate what dimension an object represents

_0d	for a point, vertex, node
_1d	for a line, edge, string
_2d	for a polygon
_3d	for a 3D surface

Like *chair_0d* for vertices of a chair or *chair_1d* for the edges of the chair. Naturally a 0d, 1d or 2d object can exist in 3D space.

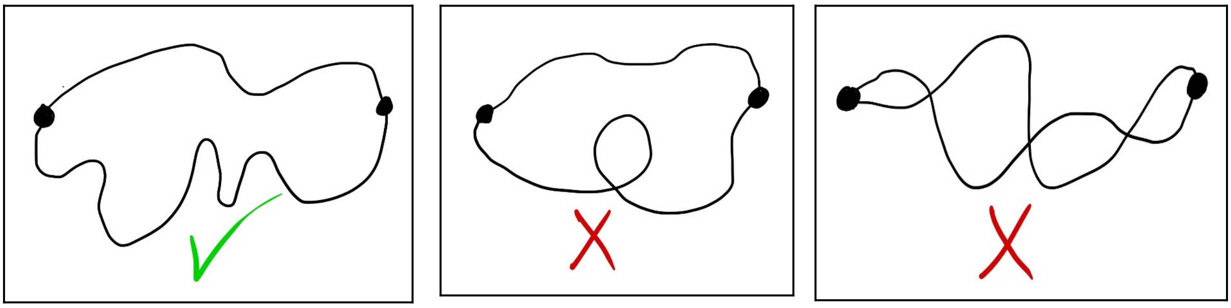


Fig-4. Polygon boundaries allowed and not allowed.

Intersections between polygons can be described at *touch*, *meet*, *overlap* and *disjoint*, but a more precise description would be making use of the type of intersection is represented. Table-1 lists these intersections of polygons and a visual presentation is shown in figure-5. The level of intersection can be formulated by the following definition “*The highest order intersection between two types of cells is equal to the lowest cell*”.

Table-1

Name	**Type Intersection**	**Abbreviation**
Disjoint	-1 cell intersection	-1_cin (minus sin)
Touch	0 cell intersection	0_cin (zero sin)
Meet	1 cell intersection	1_cin (one sin)
Overlap / Cross / Equal / Inside / Covers / Contains / Covered By	2 cell intersection	2_cin (two sin)

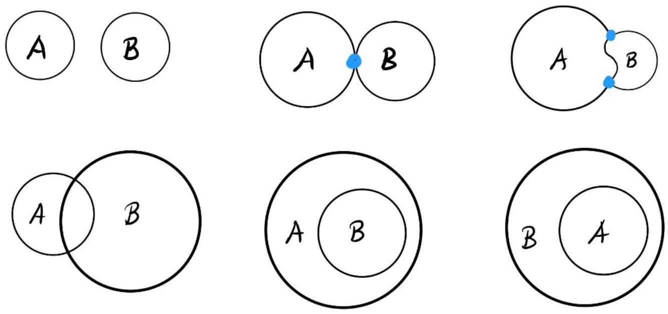


Figure-5. On top left 2 disjoint objects A & B. Top middle 2 touching objects A & B that only have a point of 0-dimension in common. The top right shows 2 polygons A & B that meet and have a 1-dimensional edge in common. At bottom A & B Overlap / Cross, are Equal or are Inside / Covering / Containing / Covered By each other.

Pattern Recognition

Humans recognise items because they see patterns associated with an object and the physical object itself; the two are linked. The author is of the opinion that recognition is based on a caricature of the object, with an underlying knowledge of its existence. That relates to the saying “ you only find what you look for”. When we recognise objects, it is because its shape but also because topological characteristics. Recognising a person that we only learned to known later in life in a childhood photo is common. It is the caricature of a person that makes it possible. Look at the caricatures of the major characters of the BBC's "Yes Minister" series (fig-6), topologically the actors' cartoons and their real images are the same. A caricatures are an essential part of pattern recognition [2]. In other words, a caricature signifies the minimum set of topological relationships to recognise an object. Because topology has no dimension but only relationships it can be represented as comprehensive graphs based on the graph theory.

Fig-6. This cartoon shows the easily recognisable main characters of the "Yes Minister" tv series.

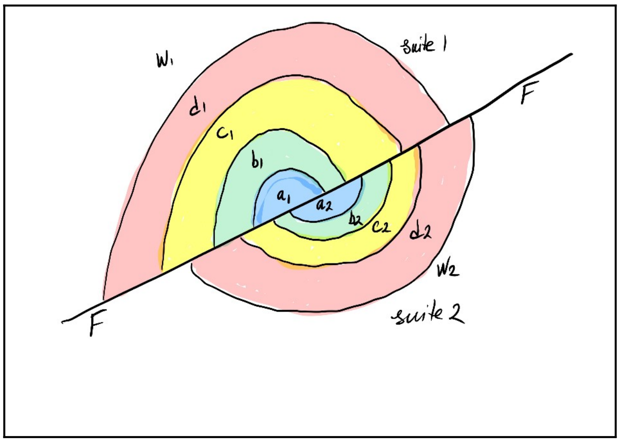
Even though geometry and geographic position are based on physical positions and shapes, the position of the objects are constrained by the topological relationship between them. In GIS topology is often used to validate objects relationships and is used when carrying out spatial queries.

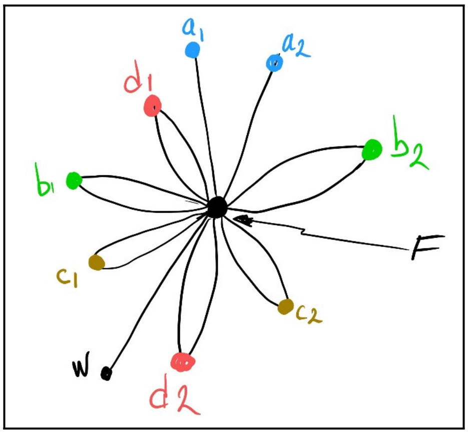
In mineral exploration when geologists try to find new prospective areas to carry out exploration for a certain commodity, they look at maps. On those maps they try to find patterns based on rocktypes and topological relationships that are typical for known type of mineralisation. In geology these topological relationships can be on meters scale, but can also occur on 100m scale, or on regional scales of 10 to 100km. Therefore, pattern recognition using graphs representing the topology could be a tool for finding new mineralisation.

Here a hypothetical example of such a geological scenario shown in figure-7. Where on the northwestern side of the fault (F) is a suite-1 of rocktypes a1, b1, c1, d1 and w1 the world outside that. On the southeastern side suite-2 with rocktypes a2, b2, c2, d2 and the outside world w2. Since the fault F is the common feature that the rocktypes abut we will show here the graph based on the F-vertex. A sketch of this scenario is shown in figure-7 a-c. The topology graph is centered around the fault shown in figure 7-b. Because the various rocktypes have contact with the fault in multiple places there are 2 connecting lines running from the vertices, e.g. from F to b2. In figure 7c also the relationship between the rocktypes is marked.

[2] The author has no idea how current facial recognition software works. So, they do not know if it uses topology.

In this example of pattern recognition in geology the suggestion is to create a topological a "*reference-graph*" graph of existing mineralisation like in figure-7b. Use this reference-graph to search maps for similar relationships.





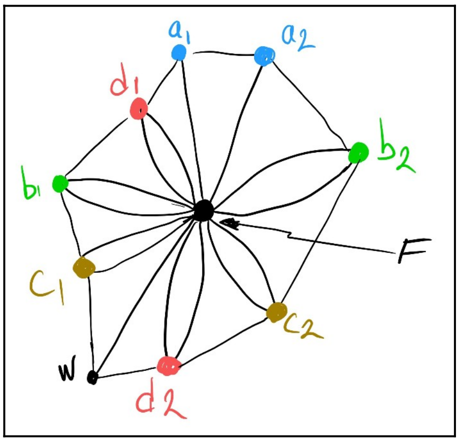


Fig7-a. Shows the spatial relationship between two suites of rocks on either side of a fault.
Fig 7-b. The topological rerelationship in graph form with the fault F at the center.
Fig 7-c. The same as 7-b but now also coneectin all rocktypes.

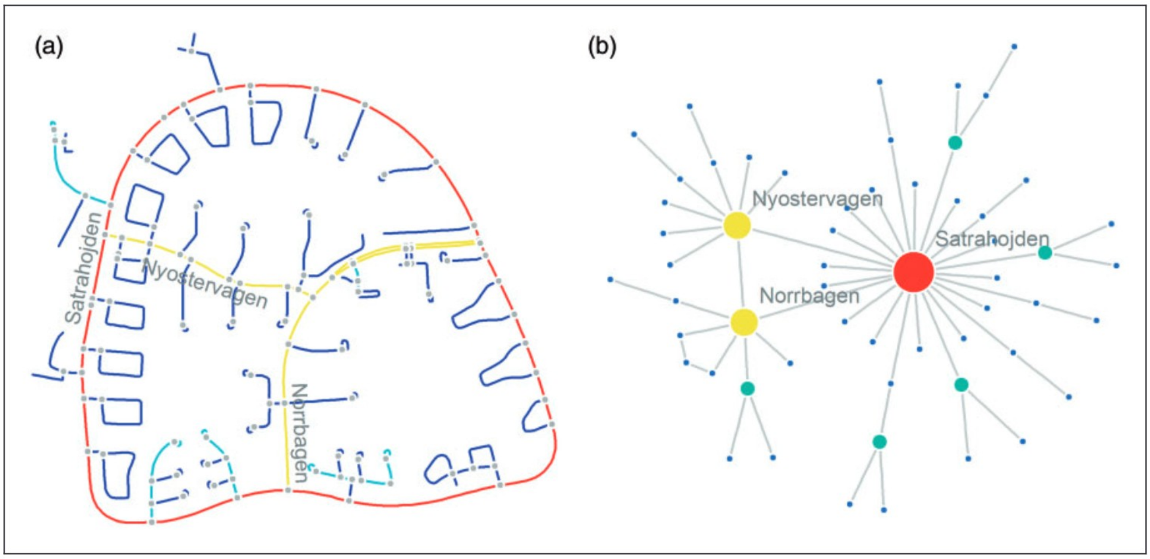


Figure 8. Transformation of geometric representation to topological representation, which enables us to see the underlying scaling of far more less-connected than well-connected streets. (a) Geometric representation, due to geometric details of locations, lengths, and directions, must be transformed into the topological repres-

entation (b) in order to see clearly the scaling property of far more less-connected streets than well-connected ones. The topological representation bears no geometric details at all. The most distinguished feature of the topological representation is its underlying scaling of numerous least-connected streets, very few most-connected ones (here the only one), and some in between the least- and most connected. Copied from Ma et al. (2019). This development is likely based on the Radburn deisgn principles.

Topological graphs can also be used for representing road hierarchies, like in figure-8. This is a visualisation of various levels , a ring road (red), internal connecting roads (yellow) and local streets (blue).

Alternative Topology for Road Center Lines

Note that what here is described is based on the centerline knowledge I acquired at a Geogeeks presentation by Ben Ritter in July 2023. Therfore the author may have made assumptions that are not valid. This whole exercise is theoretical and it practical implementation may be impossible.

One of the issues highlighted by Ritter (2023) is that when a road center line (CL) in the middle of let say a 4-lane road, splits around a median strip, the split CL runs now in the middle of the 2-lane part. This raises problems when rendering CLs to "normal" visualisation of roads on maps (fig-8). Topologically the form of the center lines is unimportant. On maps the center lines follow broadly the shape of the road.

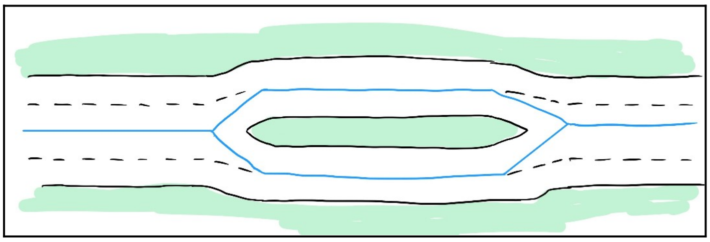


Fig-9. Here a 4-lane road splits around a median strip. The dashed lines are the marker between the two lanes going in each direction. The blue lines are the center lines.

Graph Theory is extensively used in network analysis. Platforms like Google and Uber will be using it for their routes. A lot of it will be based on approximations because many problems are intractable. The problem got first notoriety in the 1930s with the formulation of "The Traveling Salesman Problem" (Wiki, 2023-a). The problem was first formulated by Euler in 1736 when describing the "Seven Bridges of Königsberg" (Wiki, 2023-b). These problems are unlikely to be of influence when checking the topology of a road network.

Instead of using a polyline as a center line the author suggests to use polygons. In figure-10 a road is delineated by the "center line" from node N1 to N4 with intermediate nodes N2 and N3. At the top the polygons that form the center line are shown. At the bottom the areas of these polygon is collapsed to zero, so there is just a straight line. It works in QGIS, digitise vertex 1, 2 and then back to vertex 1 to close off the polygon. Following on from this setup figure-11 shows what happens when somewhere along the road where the road splits by a median strip or moves around a mountain. In geographic space P1 and P3 represent polygons with zero internal area, while P2 is a polygon covering X m^2.

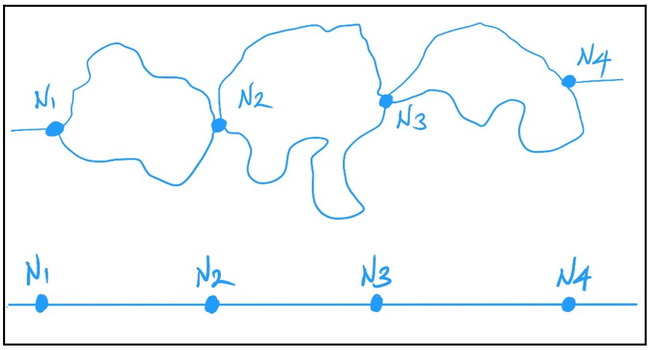


Fig-10. Relationships touching polygons form the nodes for a road center line at the top. On the bottom the same topological realtion but when the polygons are deflated to having a zero internal area.

An advantage of using a 2-vertex polygon as a representation of a line that it is easier to establish topological relationships. In GIS when topological relationships for polygons they need to have been digitised in the same direction, i.e. all in clockwise or all in anti-clockwise direction, depending of what the software requires. In an anti-clockwise digitised polygon, the righthand side is the internal space, while the lefthand side signifies its relationships with surrounding polygons.

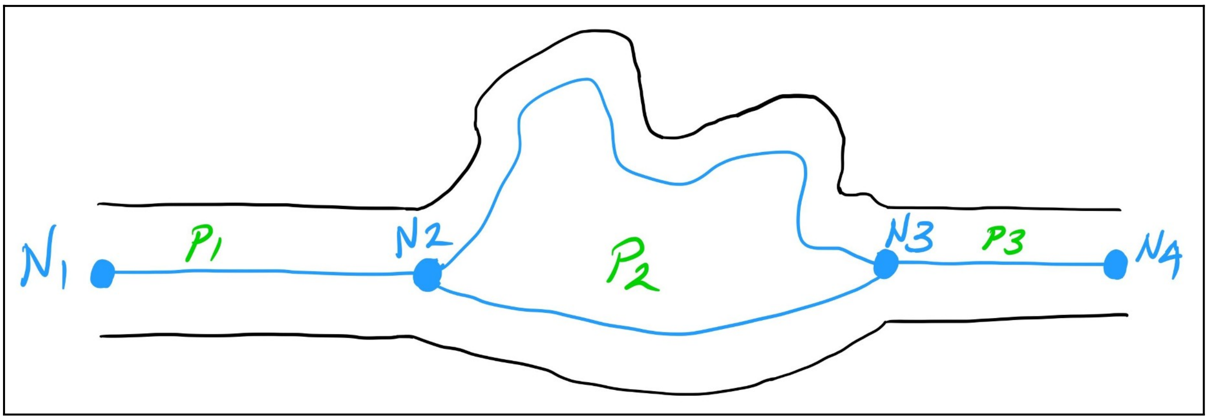


Fig 11. A road from node N1 to N4. Between N1 and N2 and N3 to N4 polygons P1 and P2 cover zero area. Polygon P2 between N2 and N3 covers an area occupied by a median strip or mountain or something else. The bluelines are the SCL.

When the centerline is “attached” to the left hand side of the polygon a fixed width can easily be assigned for the lanes representing that part of the road. Which will produce a neatly consistent width between the nodes. So if the center line is “attached” to the lefthand side of the polygon a fixed width can be assigned to it that represent the road/lane width. Which will be neatly consistent along the CL. It will be confusing to use the same term for two different types of center lines, so we propose to use “**side center line** (**SCL**)”. The SCL follows the CL when the road is “simple”, but when there is a median strip or a turning lane the SCL follows the inside curve (figure-11 and 12).

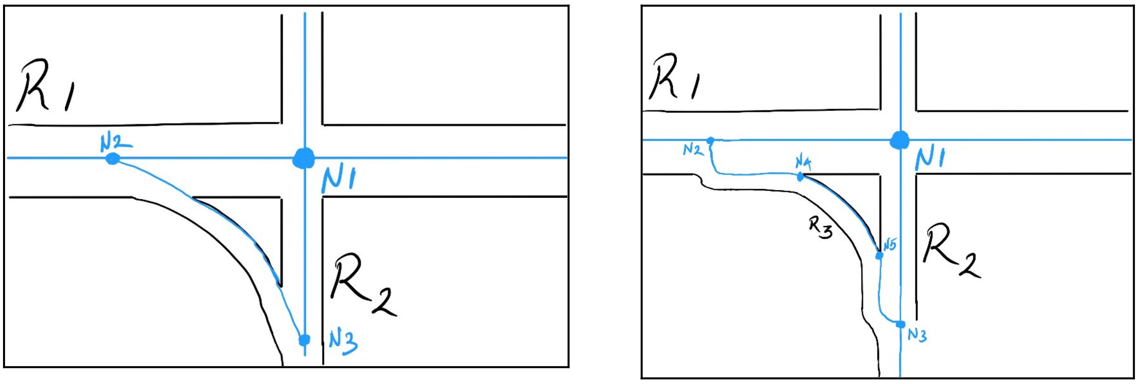


Figure-12. Two intersections where the SCL follows the edge of road.

The reason for using Nodes and Vertices for basically the same type of points is to distinguish there between locations that make up the road network center lines. Nodes define the positions where there are changes in the road, e.g. at an intersection or turnoff. While vertices are used to define the shape of the road or meet other polygons. Nodes are also forming an integral part of the network topology, while vertices are not part of the network topology.

The simple intersection of figure-13 demonstrates this. Three of the corners are quite tight while the 4^{th} has a wide curve. This create space at ther center of the intersection. Traveling along road R1 the first segment is N2 to N6, the second segment comprises of a polygon with start node N6, end node N1 and vertices V1, V2. Segment N1 to N8 is a polygon with vertices V3 and V4. The last segment is N8 to N4. The curvature of the corners are bordered by for example segments N6-V2 and V2-N9. This more complicated than when intersection have sharp corners like at node N1 in figure-12.

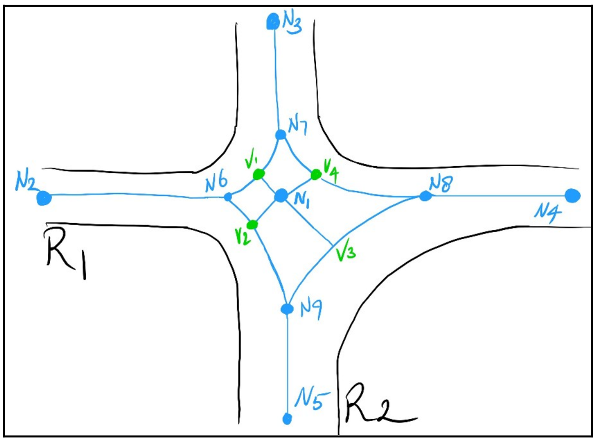


Fig-13.
This is a simple intersection where polygons are used for centerlines. That is the reason additional nodes had to be inserted at the point where the internal area became greater than zero.

What has been described in this presentation is theoretical and will most unlikely replace the underlying architecture of Openstreetmaps. But may it can be used in Apps that solve intricate problems, e.g. at intersections.

Now we have introduced zero area polygons this feature may have a use in geology. As shown in figure-14 mapping has established that part of the rock horizon containing R1 has been removed by a fault and that rocktype R2 has not been affacted by the fault. In this case the polygons covering rocktype R1 has zero width between vertices V1 and V2. It could be more useful in representing faults, which often is just done by a line but often when zooming in have a width. Possibly also makes modelling easier and more powerful since only dealing with polygons which are like in the same layer.

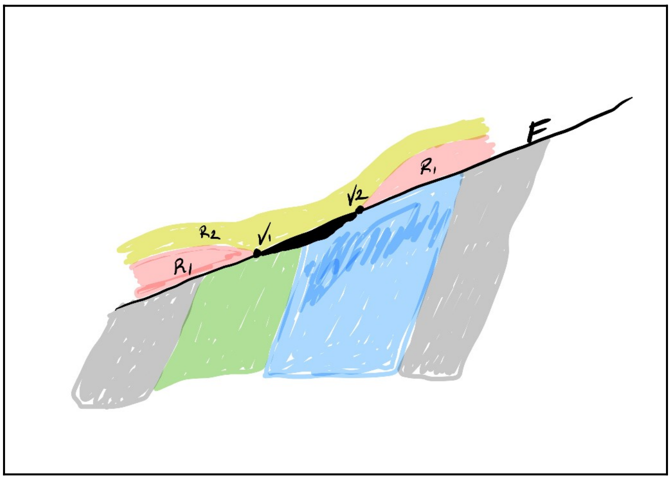


Fig-14. Rocktype R1 could have a zero width between vertices V1 and V2.

References

Boogaerdt, Humphrey, 1996. *Topology-Object-Relation-Graph*. Curtin University, thesis. www.payung.biz

Egenhofer, Max and Franzosa, Robert, 1991. *Point-set topological spatial relations*. Int J GIS. doi: 10.1080/02693799108927841

Ma, Ding., et al., 2019. *Why topology matters in predicting human activities*. Urban Analytics and City Science, 46(7), 1297–1313. https://doi.org/10.1177/2399808318792268

Ritter, Ben., 2023. *OpenStreetMap's big forking problem: OSM centrelines to full streets*. Presentation at Geogeeks, Perth.

Wiki, 2023-a. *Traveling Salesman Problem*. https://en.wikipedia.org/wiki/Travelling_salesman_problem

Wiki., 2023-b. *Seven Bridges of Königsberg*. https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg

"Yes Minister" cartoon https://socialistjazz.blogspot.com/2012_08_01_archive.html

Appendix

Side Centerline issues update

Some issues raised at the Geogeeks presentation (14 Sep 2023) regarding the Side CenterLines.

The drawings at the presentation did not cater for when a road had let say 5 lanes going in one direction. Figure-1 shows a possible solution. Upto node N3 there are 5 lanes associated with the SCL, then between N3 and N6 it will be 6 lanes. Between N5 and N4 there is jus 1 lane attached to the SCL.

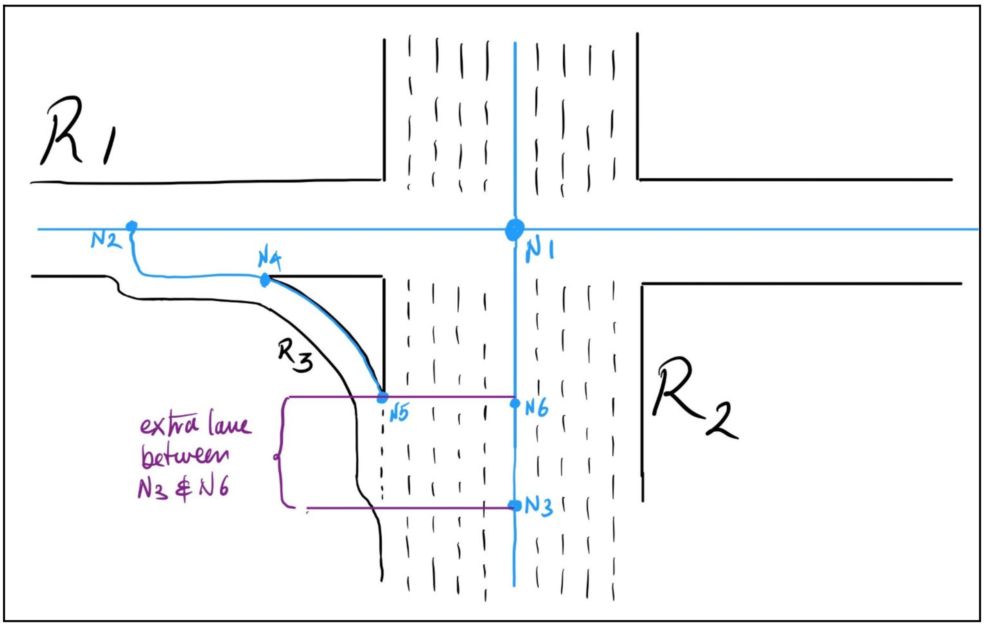


Fig-1

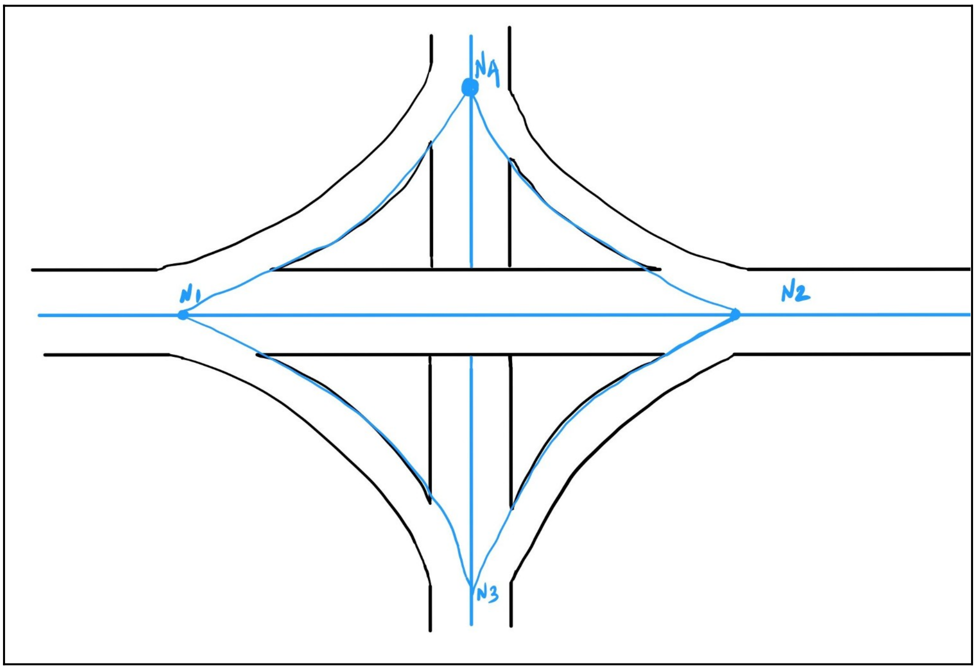
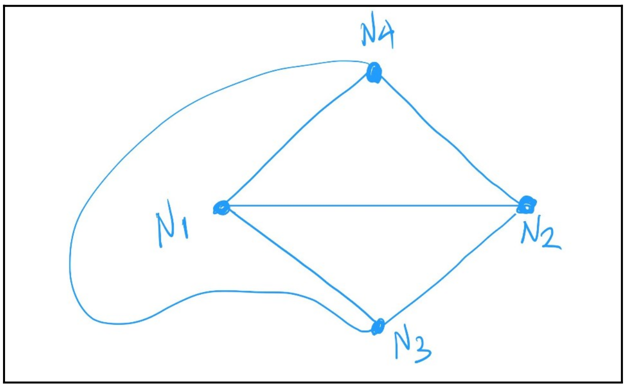


Fig-2 Bridge over road

There was the issue of topology for when a bridge goes over a road. Figure-2 shows that there is no intersection node where the bridge goes over the other road. In this scenario there are only nodes N1 to N4. The topological relationship graph is in 3D space, when displaying it in 2D space it shows like in figure-3. It can be displayed as on the left when there are simple and a few relationships. On the riight using the convention that "real" intersections are marked with a node and that crossing lines in 2D space do not insrsect.



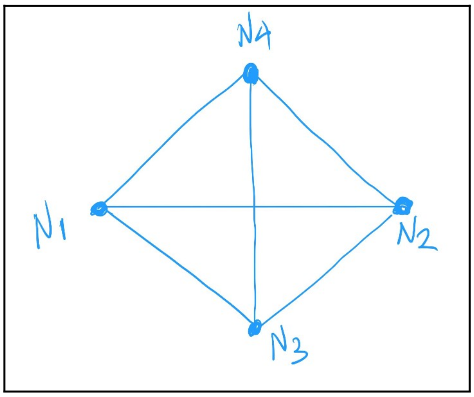


Fig-3 The topological graph of the intersection of Fig-2.